Uncertainties and Error Propagation

The general method of getting formulas for propagating errors involves the total differential of a function. Suppose that $z = f(w, x, y, \ldots)$ where the variables w, x, y, etc. must be independent variables!

The total differential is then

$$dz = \left(\frac{\partial f}{\partial w}\right) dw + \left(\frac{\partial f}{\partial x}\right) dx + \left(\frac{\partial f}{\partial y}\right) dy + \cdots$$

We treat the $dw = \Delta w$ as the error in w, and likewise for the other differentials, dz, dx, dy, etc. The numerical values of the partial derivatives are evaluated by using the average values of w, x, y, etc. The general result is

$$\Delta z = \left|\frac{\partial f}{\partial w}\right| \Delta w + \left|\frac{\partial f}{\partial x}\right| \Delta x + \left|\frac{\partial f}{\partial y}\right| \Delta y + \cdots$$

Example: Consider $S = x\cos(q)$ for $x = (2.0 \pm 0.2)$ cm, $q = (53 \pm 2)\,° = (0.9250 \pm 0.0035)$ rad. Find S and its uncertainty. Note: the uncertainty in angle must be in radians!

We have that $S = 2.0\cos(53°) = 1.204$ cm

The total differential is found by deriving *S* with respect to *x* and to θ. It leads to

$$dS = \cos(\theta)\, dx - x\, sin(\theta)\, d\theta$$

We take the absolute value of each term.

$$\Delta S = \cos(\theta)\, \Delta x + x\sin(\theta)\, \Delta\theta = 0.126 \text{ cm}$$

Hence $S = (1.20 \pm 0.13)$ cm.

Source: https://www.geol.lsu.edu/jlorenzo/geophysics/uncertainties/Uncertaintiespart2.html#addsub